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Gravitating (field theoretical) cosmic (p, q)-superstrings

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Abstract: We study field theoretical models for cosmic (p,q)-superstrings in a curved space-time. We discuss both string solutions, i.e. solutions with a conical deficit, but also so-called Melvin solutions, which have a completely different asymptotic behaviour. We show that globally regular gravitating (p,q)-strings exist only in a finite domain of the parameter space and study the dependence of the domain of existence on the parameters in the model. We find that due to the interaction between strings, the parameter range where string solutions exist is wider than for non-interacting strings.

Keywords: Topological Strings, String theory and cosmic strings.

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1. Introduction

The recent realization that some extended objects in fundamental string theory [1] can have cosmological size has triggered a lot of interest both in the fundamental string and the cosmic string community. In braneworld scenarios, big quantities of two types of cosmic size "strings" (cosmic superstrings) can be produced [2] when brane-antibrane pairs annihilate each other: fundamental (F) strings and Dirichlet (D) strings, which can be either a one dimensional brane (D1-brane) or a higher dimensional brane with all but one dimension compactified. These two kinds of strings interact with each other [3] forming bound states: p F-strings can interact with q D-strings to form (p,q) bound states, resulting in a network of F-, D- and FD-strings.

Networks of (p,q) strings have been investigated in several papers lately, both analytically and numerically, and both in the realms of fundamental strings and solitonic gauge strings [4–8]. The dynamics of a (p,q) network is more complicated than "usual" (solitonic) cosmic strings: The intercommutation of solitonic strings is almost always one, whereas in (p,q) strings it is usually less than one, and often much less than one. Generally, solitonic strings do not form bound states, so that 3-way Y junctions do not form, whereas (p,q) strings are known to form stable bound states, and therefore create Y junctions. All strings in a network of ordinary solitonic strings have the same tension, whereas there is a whole range of different tensions in a (p,q) network.

Properties of ordinary (solitonic) comics strings have been thoroughly studied, and their network properties are better understood than those of (p,q) strings. They are known

to be generically formed in supersymmetric hybrid inflation [9] and grand unified based inflationary models [10]. There has been considerable effort in numerically modelling networks to obtain observables; for example, modeling field theoretical cosmic strings to obtain CMB power spectra [11].

The gravitational properties of solitonic cosmic strings have also been studied in detail by minimally coupling the Abelian Higgs model to gravity. Far away from the core of the string, the space-time has a deficit angle, i.e. corresponds to Minkowski space-time minus a wedge [12]. The deficit angle is in linear order proportional to the energy per unit length of the string. If the vacuum expectation value (vev) of the Higgs field is sufficiently large (corresponding to very heavy strings that have formed at energies much bigger than the GUT scale), the deficit angle becomes larger than 2π. These solutions are the so-called "supermassive strings" studied in [13] and possess a singularity at a maximal value of the radial coordinate, at which the angular part of the metric vanishes. Interestingly, it was realized only a few years ago [14, 15] that both the globally regular string solution as well as the supermassive solution have "shadow" solutions that exist for the same parameter values. The string-like solutions with deficit angle $< 2\pi$ have a shadow solution in the form of so-called Melvin solutions, which have a different asymptotic behaviour of the metric and have higher energies than their string counterparts (and are thus very likely cosmologically not relevant). The supermassive string solutions on the other hand have shadow solutions of Kasner-type. Kasner solutions possess also a singularity at some finite radial distance, the difference is that for Kasner solutions, the tt-component of the metric vanishes at this maximal radial distance, while the angular part of the metric diverges there. Since both supermassive as well as Kasner solutions contain space-time singularities they are surely of limited interest for cosmological applications.

It is partly due to the fact that advanced machinery exists to study solitonic strings that different authors have tried to capture properties of (p,q) string networks using field theoretical models. One can construct field theory models that give rise to solitonic strings that share some of the properties of cosmic superstrings. For example, Abelian Higgs strings can have a reconnexion probability which is less than one when the strings meet at very high velocities [16]. Also, type-I Abelian Higgs strings can form bound states (and have strings of different tensions) when they meet at low velocities [17, 7], or if a network of (type I) strings with high winding numbers forms in a thermal phase transition [18]. In models with SUSY flat directions strings with type I properties can also be formed [19].

A maybe more realistic approximation is given by the models of Saffin [5] and Rajantie, Sakellariadou and Stoica [6], where two independent Abelian Higgs string models are coupled via an interacting potential. The strength with which the strings interact and form bound states can be controlled by the parameters in the potential, giving rise to numerically more feasible and controllable simulations, including network simulations [6, 8].

In this paper we couple minimally the models of Saffin and Rajantie et al. to gravity, and investigate the gravitating properties of this field theoretical approximations of (p,q) cosmic superstrings. In the next section we describe the models and set up the ansätze and boundary conditions. In section 3 we report the numerical solutions found for the models studied, and then conclude.

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2. The model

The models we are studying are given by the following action:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R + \mathcal{L}_m\right) \tag{2.1}$$

where R is the Ricci scalar and G denotes Newton's constant. The matter Lagrangian $\mathcal{L}_m$ reads:

$$\mathcal{L}_m = D_\mu\phi(D^\mu\phi)^* - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\xi(D^\mu\xi)^* - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} - V(\phi,\xi) \tag{2.2}$$

with the covariant derivatives $D_\mu\phi = \nabla_\mu\phi - ie_1A_\mu\phi$, $D_\mu\xi = \nabla_\mu\xi - ie_2B_\mu\xi$ and the field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ of the two U(1) gauge potential A_μ, B_μ with coupling constants e_1 and e_2. The fields ϕ and ξ are complex scalar fields (Higgs fields).

We will study two different potentials, corresponding to Saffin [5] (V_1) and Rajantie et al. [6] (V_2)

$$V_1(\phi,\xi) = \frac{\lambda_1}{4}\left(\phi\phi^* - \eta_1^2\right)^2 + \frac{\lambda_2}{4}\left(\xi\xi^* - \eta_2^2\right)^2 - \lambda_3\left(\phi\phi^* - \eta_1^2\right)\left(\xi\xi^* - \eta_2^2\right) \tag{2.3}$$

and

$$V_2(\phi,\xi) = \frac{\tilde{\lambda}_1}{4}\left(\phi\phi^* - \eta_1^2\right)^2 + \frac{\tilde{\lambda}_2}{4\eta_1^2}\phi\phi^*\left(\xi\xi^* - \eta_2^2\right)^2 \tag{2.4}$$

In the first model[1] (V_1) the interaction term makes the strings form a bound state depending on the value of $\Delta = \lambda_1\lambda_2 - 4\lambda_3^2$ [20, 5]: for $0 < \Delta < \frac{1}{2}\sqrt{\lambda_1\lambda_2}$ bounds states will be formed. In this paper we will investigate the special case where the parameters correspond to the Bogomol'nyi limit (more explicitly, to what would be the Bogomol'nyi limit if $\lambda_3 = 0$ and $G = 0$)

$$\lambda_1 = 2e_1^2 \qquad \lambda_2 = 2e_2^2 \tag{2.5}$$

and we will eventually rescale parameters such that $\eta_1 = \eta_2 = e_1 = e_2 = 1$, though we will keep them explicit throughout our equations. It is also worth reminding that by setting $\lambda_3 = 0$ we recover two non-interacting Abelian Higgs strings in curved space-time.

For the second case, the interaction ensures that it is energetically favourable to form bound states for $\tilde{\lambda}_1 > 0$ and $\tilde{\lambda}_2 > 0$ (otherwise, the potential will not be bounded from below). This is clearly seen by observing that the potential energy of the ξ field vanishes where $\chi = 0$, therefore favouring that both string cores lie on top of each other.

2.1 The Ansatz

In the following we shall analyse the system of coupled differential equations associated with the gravitationally coupled system described above. This system will contain the Euler-Lagrange equations for the matter fields and the Einstein equations for the metric fields. In order to do that, let us write down the matter and gravitational fields as shown

[1] This type of potential has also been studied in relation to composite topological defects [20, 21].

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below. The most general, cylindrically symmetric line element invariant under boosts along the $z-$direction is:

$$ds^2 = N^2(\rho)dt^2 - d\rho^2 - L^2(\rho)d\varphi^2 - N^2(\rho)dz^2 \ . \tag{2.6}$$

The non-vanishing components of the Ricci tensor R^ν_μ then read [14]:

$$R^0_0 = -\frac{(LNN')'}{N^2L} \ , \quad R^\rho_\rho = -\frac{2N''}{N} - \frac{L''}{L} \quad , \quad R^\varphi_\varphi = -\frac{(N^2L')'}{N^2L} \quad , \quad R^z_z = R^0_0 \tag{2.7}$$

where the prime denotes the derivative with respect to ρ.

For the matter and gauge fields, we have [22]:

$$\phi(\rho,\varphi) = \eta_1 h(\rho) e^{in\varphi} \, , \tag{2.8}$$

$$\xi(\rho,\varphi) = \eta_1 f(\rho) e^{im\varphi} \, , \tag{2.9}$$

$$A_\mu dx^\mu = \frac{1}{e_1}(n - P(\rho))d\varphi \ . \tag{2.10}$$

$$B_\mu dx^\mu = \frac{1}{e_2}(m - R(\rho))d\varphi \ . \tag{2.11}$$

n and m are integers indexing the vorticity of the two Higgs fields around the $z-$axis.

2.2 Equations of motion

We define the following dimensionless variable and function:

$$x = e_1\eta_1\rho \quad , \quad L(x) = \eta_1 e_1 L(\rho) \ . \tag{2.12}$$

Then, the total Lagrangian only depends on the following dimensionless coupling constants

$$\gamma = 8\pi G\eta_1^2 \ , \ g = \frac{e_2}{e_1} \ , \ q = \frac{\eta_2}{\eta_1} \ , \ \beta_i = \frac{\lambda_i}{e_1^2} \, , \ i = 1,2,3 \ \text{ or } \ \tilde{\beta}_i = \frac{\tilde{\lambda}_i}{e_1^2} \, , \ i = 1,2 \ . \tag{2.13}$$

Varying the action with respect to the matter fields and metric functions, we obtain a system of six non-linear differential equations. The Euler-Lagrange equations for the matter field functions read:

$$\frac{(N^2Lh')'}{N^2L} = \frac{P^2h}{L^2} + \frac{1}{2}\frac{\partial V_i}{\partial h} \, , \ i = 1,2 \tag{2.14}$$

$$\frac{(N^2Lf')'}{N^2L} = \frac{R^2f}{L^2} + \frac{1}{2}\frac{\partial V_i}{\partial f} \, , \ i = 1,2 \tag{2.15}$$

$$\frac{L}{N^2}\left(\frac{N^2P'}{L}\right)' = 2h^2P \, , \tag{2.16}$$

$$\frac{L}{N^2}\left(\frac{N^2R'}{L}\right)' = 2g^2f^2R \, , \tag{2.17}$$

where the prime now and in the following denotes the derivative with respect to x.

We use the Einstein equations in the following form:

$$R_{\mu\nu} = -\gamma \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \quad , \quad \mu, \nu = t, x, \varphi, z \tag{2.18}$$

where T is the trace of the energy momentum tensor $T = T^{\lambda}_{\lambda}$ and the non-vanishing components of the energy-momentum tensor are (we use the notation of [14]) with $i = 1, 2$:

$$\begin{aligned} (T^0_0)_i &= \epsilon_s + \epsilon_v + \epsilon_w + u_i \\ (T^x_x)_i &= -\epsilon_s - \epsilon_v + \epsilon_w + u_i \\ (T^{\varphi}_{\varphi})_i &= \epsilon_s - \epsilon_v - \epsilon_w + u_i \\ (T^z_z)_i &= (T^0_0)_i \end{aligned} \tag{2.19}$$

where

$$\epsilon_s = (h')^2 + (f')^2 \quad , \quad \epsilon_v = \frac{(P')^2}{2L^2} + \frac{(R')^2}{2g^2 L^2} \quad , \quad \epsilon_w = \frac{h^2 P^2}{L^2} + \frac{R^2 f^2}{L^2} \tag{2.20}$$

and

$$\begin{aligned} u_1 &= \frac{\beta_1}{4} \left(h^2 - 1\right)^2 + \frac{\beta_2}{4} \left(f^2 - q^2\right)^2 - \beta_3 \left(h^2 - 1\right) \left(f^2 - q^2\right) \quad \text{or} \\ u_2 &= \frac{\tilde{\beta}_1}{4} \left(h^2 - 1\right)^2 + \frac{\tilde{\beta}_2}{4} h^2 \left(f^2 - q^2\right)^2 \end{aligned} \tag{2.21}$$

corresponding to the choice of potential V_1 and V_2, respectively.

We then obtain

$$\frac{(LNN')'}{N^2 L} = \gamma \left[\frac{(P')^2}{2L^2} + \frac{(R')^2}{2g^2 L^2} - u_i \right] , \; i = 1, 2 \tag{2.22}$$

and:

$$\frac{(N^2 L')'}{N^2 L} = -\gamma \left[\frac{2h^2 P^2}{L^2} + \frac{2R^2 f^2}{L^2} + \frac{(P')^2}{2L^2} + \frac{(R')^2}{2g^2 L^2} + u_i \right] , \; i = 1, 2 \tag{2.23}$$

2.3 Boundary conditions

The requirement of regularity at the origin leads to the following boundary conditions:

$$h(0) = 0, \; f(0) = 0 \,, \; P(0) = n \,, \; R(0) = m \tag{2.24}$$

for the matter fields and

$$N(0) = 1, \; N'(0) = 0, \; L(0) = 0 \,, \; L'(0) = 1 \; . \tag{2.25}$$

for the metric fields.

In the special case where $n \neq 0$ and $m = 0$ the boundary conditions (2.24) change according to:

$$h(0) = 0, \; f'(0) = 0 \,, \; P(0) = n \,, \; R(0) = 0 \,. \tag{2.26}$$

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while for a $n = 0$, $m \neq 0$ string, they read

$$h'(0) = 0,\ f(0) = 0\,,\ P(0) = 0\,,\ R(0) = m\ . \tag{2.27}$$

By letting the derivatives of the non-winding scalar field be zero at the origin instead of imposing the boundary conditions for the fields themselves, the non-winding scalar field can take non-zero values at the origin and, as it is indeed the case, a "condensate" of the non-winding scalar field will form in the core of the winding string.

The finiteness of the energy per unit length requires:

$$h(\infty) = 1,\ f(\infty) = q\,,\ P(\infty) = 0\,,\ R(\infty) = 0\ . \tag{2.28}$$

We define as inertial mass per unit length of the solution

$$E_{\rm in} = \int \sqrt{-g_3} T_0^0 dx d\varphi \tag{2.29}$$

where g_3 is the determinant of the $2+1$-dimensional space-time given by (t, x, φ). This then reads:

$$E_{\rm in} = 2\pi \int_0^\infty NL\,(\epsilon_s + \epsilon_v + \epsilon_w + u_i) \tag{2.30}$$

As mentioned earlier, for Saffin's model V_1, $\beta_3 = 0$ corresponds to two non-interacting strings. In the BPS limit (the limit we are dealing with) the inertial mass (in units of 2π) of the $(1,1)$ string is just $E_{\rm in}(\gamma) = 2$. This is because the mass is equal to the sum of the masses of the $(1,0)$-string and of the $(0,1)$-string, which is $E_{\rm in}^{(1,0)}(\gamma) = E_{\rm in}^{(0,1)}(\gamma) = 1$. Correspondingly, the deficit angle (in units of 2π) is just $\delta/(2\pi) = 2\gamma$.

We can then define the binding energy of an (n, m) string as

$$E_{\rm bin}^{(n,m)} = E_{\rm in}^{(n,m)} - nE_{\rm in}^{(1,0)} - mE_{\rm in}^{(0,1)} \tag{2.31}$$

3. Numerical results

3.1 Generalities

The solutions for $\gamma = 0$, i.e. in flat space-time have been discussed in [5] and [6]. When γ is increased from zero, the (p,q)-strings get deformed by gravity. If γ is not too large, globally regular solutions exist. These solutions fall into two categories [14, 15]: the string solutions and the Melvin solutions, where the latter have no flat-space counterpart. The string solutions are those of astrophysical interest since they describe solutions with a deficit angle and because the Melvin solutions are heavier than their string counterparts. The metric functions then have the following behaviour at infinity:

$$N(x \to \infty) \to c_1\ \ ,\ \ \ L(x \to \infty) \to c_2 x + c_3\ \ ,\ \ \ c_2 > 0\,, \tag{3.1}$$

where the c_i, $i = 1, 2, 3$ are constants that depend on the choice of γ, β_i, $i = 1, 2, 3$, q and g. The solution has a deficit angle which is given by

$$\delta = 2\pi(1 - c_2) \tag{3.2}$$

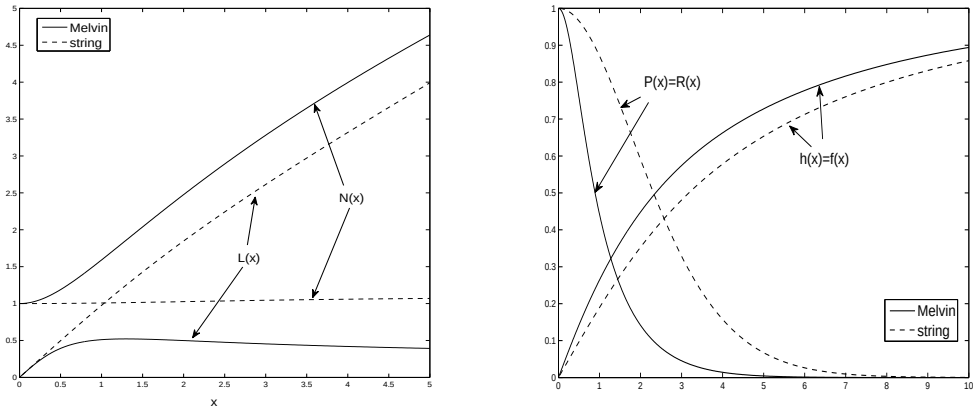

Figure 1: The profiles of the metric functions $N(x)$ and $L(x)$ (left) and of the matter functions $P(x) = R(x)$, $h(x) = f(x)$ (right) are shown for a Melvin solution (solid) and a string solution (dashed). Here $\beta_1 = \beta_2 = 2$, $\beta_3 = 0.99$, $q = g = 1$, $\gamma = 0.5$ and $n = m = 1$.

If the deficit angle $\delta > 2\pi$, $c_2 < 0$ and the solution would have a singularity at a finite, parameter-dependent value of $x = x_0$ with $L(x = x_0) = 0$, while $N(x_0)$ stays finite. These solutions are the so-called supermassive string solutions [13] (or inverted string solutions).

The Melvin solutions exist for the same parameter values as the string solutions, but have a different asymptotic behaviour:

$$N(x \to \infty) \to a_1 x^{2/3} \ , \quad L(x \to \infty) \to a_2 x^{-1/3} \ , \tag{3.3}$$

where again a_1 and a_2 are parameter dependent constants. This latter solution is thus not a cosmic string solution in the standard sense since it corresponds to a solution for which the circumference of circles will decay in the asymptotic region.

The profiles of typical string and Melvin solutions are shown in figure 1.

3.2 String solutions for the potential V_1

We have solved the above system of equations for $\beta_i = 2$, $i = 1, 2$, $g = 1$ (which for $\gamma = 0$ corresponds to the BPS limit) and $q = 1$. First, we have fixed $\gamma = 0.5$ and investigated the dependence of the deficit angle on the binding parameter β_3 and on the choice of n and m. Note that in order for $h(x) = 1$, $f(x) = 1$ to be the global minimum of the theory for our choice of the remaining parameters given above, we have to require $0 < \beta_3 < 1$.

Our numerical results are given in figure 2. The deficit angle is decreasing for increasing β_3 and tends to zero in the limit $\beta_3 \to 1$. This is expected, since the increase of β_3 leads to a stronger binding between the strings, the total energy decreases and thus results in a smaller deficit angle. Moreover, the deficit angle for a fixed value of β_3 is smaller for an (n, n) string than for an $(n + 1, n - 1)$ string, which itself is smaller than that for an $(n + 2, n - 2)$ string etc. This is clearly seen for the deficit angle of the $(2, 0)$ string in comparison to that of the $(1, 1)$ string and accordingly for strings with $n + m = 3$ and $n + m = 4$, respectively.

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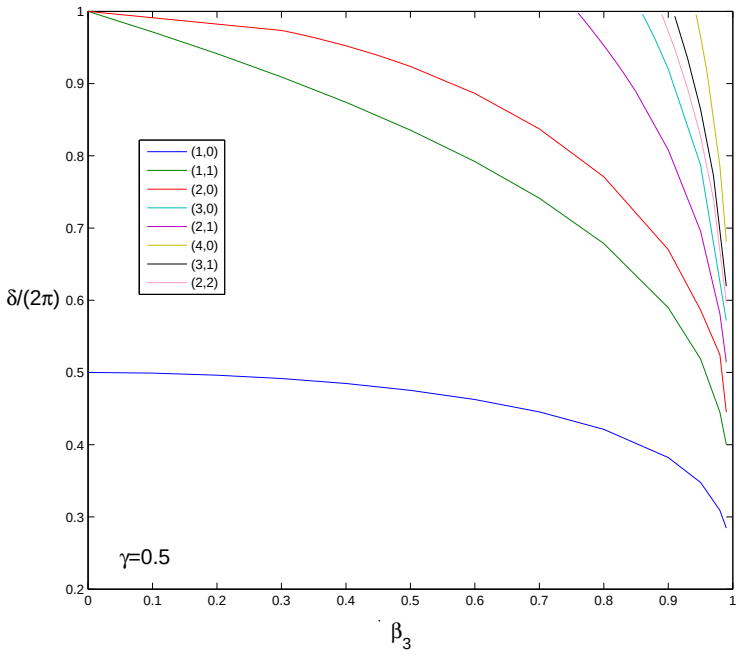

Figure 2: The value of the deficit angle δ (in units of 2π) is shown as function of β_3 for $\gamma = 0.5$ and different values of (n, m).

It is also obvious from figure 2 that globally regular string solutions exist only for a specific parameter range. For $\beta_3 = 0$, the deficit angle is given by $\delta/(2\pi) = (n+m)\gamma$. Since globally regular string solutions exist only for $\delta \leq 2\pi$, we need $(n+m)\gamma \leq 1$ or for $\gamma = 0.5$, $n + m \leq 2$. Thus, for our specific choice of γ and for $\beta_3 = 0$ we would not expect regular string solutions to exist for $n + m \geq 3$. This is clearly seen in the figure, where string solutions with deficit angle less than 2π exist for all values of β_3 only for the $(1, 0)$, $(1, 1)$ and $(2, 0)$ cases. However, regular string solutions exist also for $n + m \geq 3$ if β_3 is large enough. For $n + m \geq 3$ and a fixed value of γ, regular string solutions exist for $\beta_3 > \beta_3^{(cr)}(\gamma, n, m)$. We find e.g. $\beta_3^{(cr)}(0.5, 2, 1) \approx 0.76$, $\beta_3^{(cr)}(0.5, 3, 0) \approx 0.86$, $\beta_3^{(cr)}(0.5, 2, 2) \approx 0.89$ and $\beta_3^{(cr)}(0.5, 4, 0) \approx 0.94$. The value $\beta_3^{(cr)}$ increases for $n + m$ increasing, since a higher total winding requires a higher binding energy in order to render the string not too heavy. Moreover, for $(n, m) = (n + k, n - k)$ strings, where $0 \leq k \leq n$, the value of $\beta_3^{(cr)}$ increases for increasing k.

We have also studied the dependence of the deficit angle on the gravitational coupling for a fixed value of $\beta_3 = 0.95$. The results are given in figure 3.

As expected the deficit angle increases for increasing γ and tends to 2π for $\gamma \to \gamma^{(cr)}(\beta_3, n, m)$. Thus, globally regular strings exist only for $\gamma \leq \gamma^{(cr)}(\beta_3, n, m)$. This critical value of the gravitational coupling decreases for $n + m$ increasing and for a fixed value of $n + m$ and $(n, m) = (n + k, n - k)$, $0 \leq k \leq n$, decreases for k increasing.

Clearly, the specific choice of β_3, γ and (n, m) determines whether globally regular

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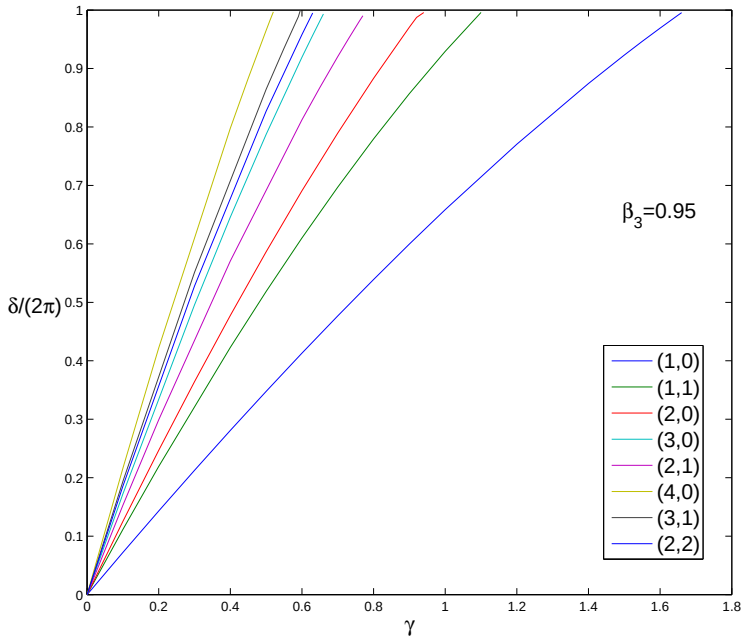

Figure 3: The value of the deficit angle δ (in units of 2π) is shown as function of γ for $\beta_3 = 0.95$ and different values of (n, m).

solutions with deficit angle smaller than 2π exist. Increasing β_3 leads to a stronger binding between the strings, i.e. the energy of the (n, m)-string is lowered and hence globally regular solutions exist for higher values of γ. This is demonstrated in figure 4 for $(1, 0)$, $(1, 1)$ and $(2, 1)$ strings. Here, we plot the value of the gravitational coupling up to which the regular solutions exist, $\gamma^{(cr)}$, as function of β_3. Note that regular solutions exist only in the parameter domain below the respective curves. It is clear that the solutions exist for higher values of γ when β_3 is increased. This is related to the increase in binding energy, e.g. for the $(2, 1)$ solution with $\gamma = 0.5$, we find that $E_{\rm bin}^{(2,1)} \approx -0.551$ for $\beta_3 = 0.8$, while $E_{\rm bin}^{(2,1)} \approx -0.652$ for $\beta_3 = 0.95$. In addition to the binding due to the potential, there is an additional binding due to gravity. The larger γ, the stronger is the binding: e.g. for $\beta_3 = 0.9$, we find $E_{\rm bin}^{(2,1)} \approx -0.621$ for $\gamma = 0.5$, while $E_{\rm bin}^{(2,1)} \approx -0.581$ for $\gamma = 0$, in agreement with what it has been reported [5].

3.3 String solutions for the potential V_2

First, we have studied the solutions for $\tilde{\beta}_1 = \tilde{\beta}_2 = 1$, $q = g = 1$ and $\gamma = 0.5$ to understand what qualitative differences appear in comparison to the potential V_1. While in the case of the V_1 potential (and in the BPS limit) a $(1, 0)$-string is exactly equivalent to a $(0, 1)$ string, this is different here. As in the V_1 case, when one string is winding and the other is not, the non-winding string can form a condensate at the core of the other string. However, in this case, a condensate of the ξ field in the core of a ϕ string does not lower the energy; whereas

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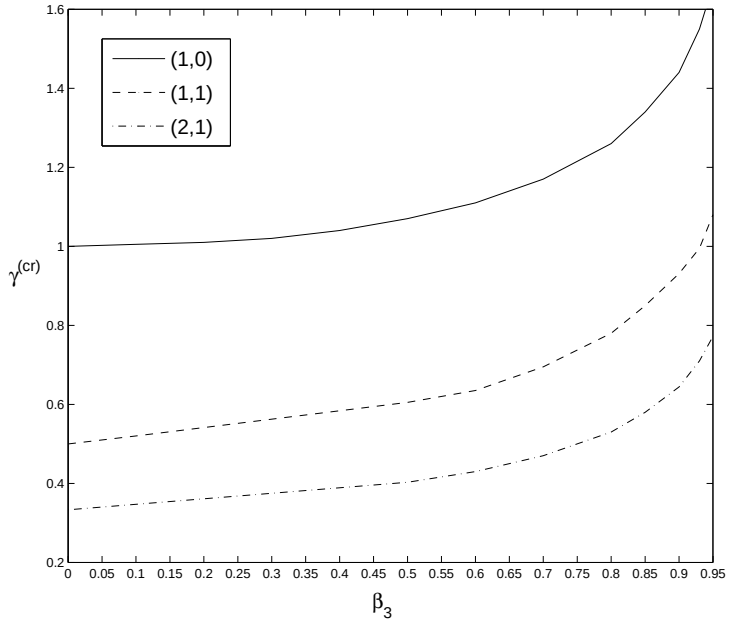

Figure 4: The value of $\gamma^{(cr)}$ is shown as function of β_3 for $(1,0)$, $(1,1)$ and $(2,1)$ strings. Note that globally regular solutions for the specific choices of (n,m) exist only in the parameter domain below the corresponding curves.

a condensate of the ϕ inside the ξ core does. More specifically, for $n=1$, $m=0$, the scalar field function $f(x)$ is not forced to zero at the origin and could develop a condensate. However, $f(x) \equiv q$ everywhere and correspondingly $R(x)=0$ gives the minimum for the potential for the boundary conditions we have. On the other hand, for $n=0$ and $m=1$, the scalar field function $h(x)$ is also not forced to zero at the origin, but does develop a condensate. At the origin, a value of $h(x)<1$ is the value that minimizes the energy best. Thus, a $(1,0)$-string is different from a $(0,1)$ string. This can also be seen when looking at the numerical values of the energies and deficit angles. For $(1,0)$, we find $E_{\rm in} \approx 0.859$ and $\delta/(2\pi) \approx 0.425$, while for $(0,1)$, we have $E_{\rm in} \approx 0.848$ and $\delta/(2\pi) \approx 0.418$ (see also figure 7).

In figure 5 we show the dependence of the deficit angle on the gravitational coupling γ. This is qualitatively very similar to the case with the V_1 potential. Again, we observe that globally regular string like solutions exist only up to a maximal value of the gravitational coupling $\gamma = \gamma^{(cr)}(n,m)$. If one would increase γ further, the solutions would have a deficit angle $> 2\pi$ and would thus be singular. $\gamma^{(cr)}(n,m)$ decreases for $n+m$ increasing and for a fixed value of $n+m$ and $(n,m)=(n+k,n-k)$, $0 \le k \le n$, decreases for k increasing.

As for the V_1 case, the specific choice of β_3, γ and (n,m) determines, whether globally regular solutions exist. The fact that the strings interact and form bound states, lowers their energy. Therefore, for values of γ and (n,m) where one would not expect regular global solution in the non-interacting case, regular solutions exists when strings interact.

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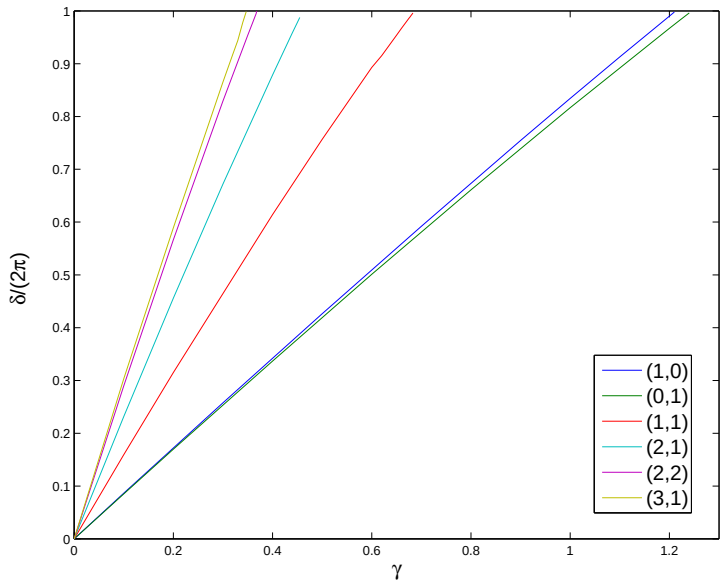

Figure 5: The value of the deficit angle δ (in units of 2π) is shown as function of γ for $\tilde{\beta}_1 = \tilde{\beta}_2 = 1$, $q = g = 1$ and different values of (n, m).

3.4 Melvin solutions

We have also studied the corresponding Melvin solutions for both potentials. Like their string counterparts, they exist only in a limited domain of the parameter plane.

In the V_1 case, for a fixed value of γ, $\beta_3^{(cr)}$ is equal for the string and Melvin solutions. This has already been observed in the case of a single gravitating string [14, 15]. At this $\beta_3^{(cr)}$, the inertial energies $E_{\rm in}$ of the two types of solutions become equal. This is shown in figure 6 for $(n, m) = (3, 0)$ and $(n, m) = (2, 1)$, respectively. At the critical value of $\beta_3^{(cr)}$ the curves of the inertial energies of the two types of solutions merge. Note also that for all cases studied in this paper, the inertial energy of the Melvin solutions is larger than that of the corresponding string solutions.

For $\beta_3 < \beta_3^{(cr)}$, the string solutions become inverted/supermassive string solutions with a zero of the function $L(x)$ at some parameter dependent value of $x = x_0$, while $N(x = x_0)$ stays finite. The Melvin solutions - on the other hand - become so-called Kasner solutions with $N(x = \tilde{x}_0) = 0$ and $L(x \to \tilde{x}_0) \to \infty$ at some finite and parameter-dependent value of $x = \tilde{x}_0$. The inverted string and Kasner solutions thus exist only on a finite range of the coordinate x and are thus sometimes also called "closed" solutions. These solutions have singularities in their metric, and it is out of the scope of this paper to investigate their implications.

We have also studied the Melvin solutions in the case of V_2. Again, the qualitative results are very similar to the case with V_1. In figure 7, we show the dependence of the inertial energy $E_{\rm in}$ on the gravitational coupling γ for $\tilde{\beta}_1 = \tilde{\beta}_2 = 1$, $q = g = 1$ and three

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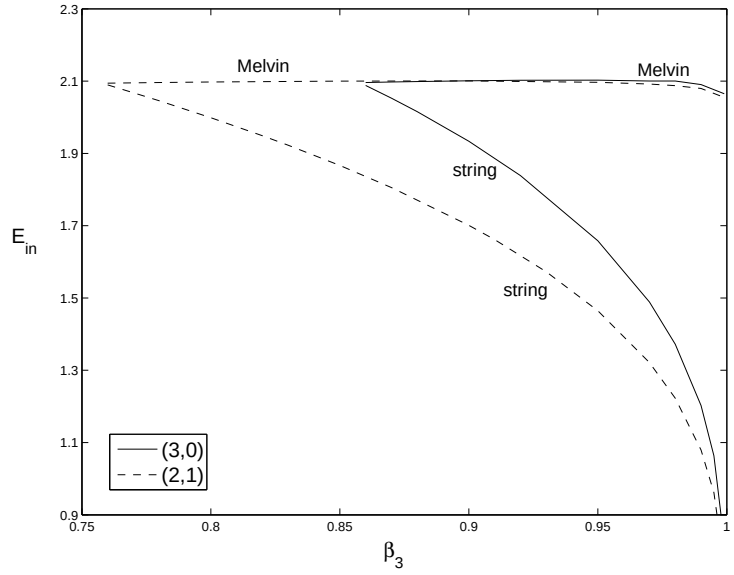

Figure 6: The dependence of the inertial energy $E_{\rm in}$ on the binding parameter β_3 is shown for string and Melvin solutions with $(n, m) = (3, 0)$ (solid) and $(n, m) = (2, 1)$ (dashed). Here $\beta_1 = \beta_2 = 2$, $q = g = 1$ and $\gamma = 0.5$.

different choices of (n, m). The inertial energy of the string solutions is always lower than that of the Melvin solutions and the two branches of solutions meet at $\gamma^{(cr)}$, the maximal value of the gravitational coupling, beyond which no globally regular solutions exist. As before, the maximal value of the gravitational coupling decreases with $n + m$ increasing. We include both $(1, 0)$ and $(0, 1)$ solutions, since, as stated above, the do differ for the V_2 case.

It has been observed previously [15] that the inertial energy of the Melvin solutions doesn't depend strongly on the winding number. The reason for this is that the main contribution to the energy of the Melvin solution comes from the gravitational field and is thus "insensitive" to the actual core structure of the solution. This explains why the energy of the Melvin solution practically doesn't depend on β_3 (see figure 6), while it has a strong dependence on the gravitational coupling (see figure 7). One could see these effects by studying the gravitationally active mass, the so-called Tolman mass $M_{\rm tol}$ [23], which for our definition of the metric is given by [14]: $M_{\rm tol} \propto \lim_{x\to\infty}(LNN')$. Clearly, for the standard string solutions, this mass is zero, while for the Melvin solutions it is non-vanishing. The Tolmann mass of the Melvin solutions would then depend on the windings of the strings (see also the discussions in [15]).

4. Conclusions and discussion

In this paper we have studied the gravitating properties of strings arising in field theoretical approaches to (p,q)-superstrings, by minimally coupling the field theoretical models to

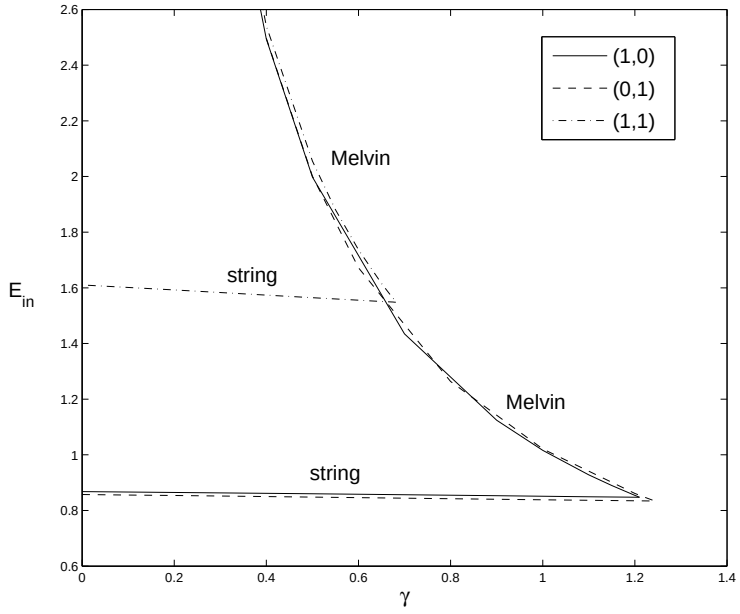

Figure 7: The dependence of the inertial energy $E_{\rm in}$ on the gravitational coupling γ is shown for string and Melvin solutions in the case of V_2 with $\tilde{\beta}_1 = \tilde{\beta}_2 = 1$, $q = g = 1$. Here, we compare the cases $(n, m) = (1, 0)$ (solid), $(n, m) = (0, 1)$ (dashed) and $(n, m) = (1, 1)$ (dotted-dashed).

gravity. We find that there exist globally regular solutions of both the string type (solutions with conical deficit) and the Melvin type for a finite domain in parameter space.

The existence of globally regular solutions depends very strongly on the (inertial) energy of the underlying string specimen. Therefore, the specific choices of windings of the strings and the string interaction (as well as obviously the strength of the gravitational interaction) will determine whether regular solutions exists. Due to the interaction term, the inertial energy of a (p,q) string is lower than the sum of the inertial energy of its constituents; thus, the parameter region where regular strings exist gets enlarged with a stronger interaction term. We also found that it is not only the interaction between strings which makes the binding stronger, but a stronger gravitational interaction increases the binding further.

Besides string-like solutions, we have also studied Melvin solution for these models. Like their string counterparts, they exist for a finite range in parameter space, and the range gets enlarged when the interaction between strings gets stronger. There is some critical value of the parameters beyond which no regular solution exists. At that critical value, the inertial energy of the Melvin and the string solution coincide, i.e. the solutions are degenerate. For regular solutions at values of the parameters which are not critical, the Melvin solution is always heavier than the string solution.

While we have studied the situation where we have straight infinitely long strings, it would be very interesting to study the situation where three segments of strings meet in

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a Y junction. The gravitational lensing of such a Y junction has been studied for cosmic superstrings using several approximations [24]. It would be interesting to see whether we can learn something from the full study of these Y-junctions in field theory, and how they could be translated to proper cosmic superstrings.

Another interesting situation would be the following: suppose that we have two strings with winding numbers such that their regular solution exists for a given set of parameters. Let us suppose also that the parameters are such that when these two strings interact to form a new bound string, there is no regular solution of these new string. Then, we would have formed a Y junction in which one of the legs is a "supermassive string" with a zero of the metric at some finite value of the radial coordinate. This situation would be worth studying.

There are other extensions of this work which deserve being studied. Even though we are dealing with "extensions" of the Abelian Higgs model here, string solutions in non-Abelian models have also been discussed [25]. It would be interesting to study the gravitational effects of the bound states in such models.

Acknowledgments

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